FEDERAL UNIVERSITY LOKOJA
KOGI STATE

INAUGURAL LECTURE

# MATHEMATICAL FIXATION ALGORITHM FOR CORRUPTION

BY

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FEDERAL UNIVERSITY LOKOJA
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICAL SCIENCES
LOKOJA, KOGI STATE, NIGERIA

JULY, 2018

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The Second Inaugural Lecture delivered under the Chairmanship of

The

Vice Chancellor

Professor Angela Freeman Miri

B. A (UNIMAID) M. A. Ph.D (UNIJOS) ADLSCM (UNIJOS)

JULY 2018

PROFESSOR JOSEPH OLORUNJU OMOLEHIN
B.Ed(ABU), M.Sc (Unilorin), P.hD. (Unilorin) FAC
Professor of Mathematics

The Vice Chancellor,

Deputy Vice Chancellor,

The Registrar,

The Bursar,

Deans of Faculties,

Directors of Units,

Professors and Other members of Senate,

Other members of Academic staff,

Members of Administrative and Technical staff of this Great University,

My Lords Spiritual and Temporal,

Members of my Nuclear and Extended family,

Distinguished Invited Guest,

Great FULITES,

Gentle men of Print and Electronic Media,

Ladies and Gentlemen

## 1. PREAMBLES TO FULL CHAIR

Glory be to God Almighty for giving me the opportunity to live up till now to deliver this inaugural lecture.

The Devine power of God has made me to reach the apex of the carrier given to me by Him. I have passed through the shadow of death several times but the Almighty God still spare my life up till today. Madam. Vice Chancellor Ma, my becoming a Mathematician is a Devine act of God. This is because I did not attend any secondary school throughout my live. After my primary education I was not opportune to go a secondary school. When I was in the primary school I was leading in my class educationally and I was the best pupil in Arithmetic. Infarct I used to assist my colleagues to do their homework (Assignment). After my primary education I returned to the farm to assist my parents. At my leisure time, usually on weekends, I used to practice mathematics questions contained in our primary mathematics text book called "LACOMBE" and I was getting the answers correctly. I then realized that if given the opportunity to attend a secondary school I would perform very well especially in Mathematics. I believed that one day, by the Grace of God and hard work I would achieve my ambition of getting higher education..

My journey to becoming a Mathematician started in 1971 when my Uncle, Alhaji (Chief) D. O Bakare, took me to Zaria and facilitated apprentice work for me at Central Mechanical Workshop , Estate Department, ABU Zaria. In 1973, I used the stipend received during my apprenticeship to enrolled for a correspondence study with a body known as ‘’Exam Success’’. I took University of Cambridge GCE’s ‘O’ level in 1975 and I passed all my papers at Credit level. I got admission to read Diploma in Mathematics Education at ABU Zaria in 1975. While I was doing my Diploma in Mathematics Education, I enrolled for ‘A’ level in Economics and Geography which I passed.

Summarily, my journey to Professor of Mathematics is as follows:

| | | |
|---|---|---|
| i. | Diploma in Mathematics Education | (ABU,1977) |
| ii. | Higher Diploma in Mathematics Education | (ABU, 1980) |
| iii. | B.Ed Mathematics Education | (ABU, 1984) |
| iv. | M.Sc Mathematics | (Unilorin, 1986) |
| v. | Ph.D. Mathematics | (Unilorin, 1991) |
| vi. | Postdoctoral Research in Mathematics | (Leeds, UK, 1995) |
| vii. | Professor of Mathematics | (KWASU, 2009) |

I give glory to his Holiness for making me a Mathematician. This is just to inform you that there is nothing impossible with God .

Madam. Vice Chancellor Ma, American Mathematical Society is the highest Authority on Mathematics on earth. It has a section called Mathematical Review (MR) where some of my published research works in mathematics are further reviewed and indexed by experts in the field. For complete detail just go to MR section of the website of American Mathematical Society and type Omolehin you will see some of my research papers there. I am not only a local researcher but an international scholar. This is a further justification for my promotion to the full chair.

## 2. INAUGURAL AND VALEDICTORY LECTURES

One of the most important functions of University system is inaugural lecture. It is a basis for which the lecturer is promoted to full chair. The lecturer is expected to show case his area of research with a view to collaborating with other researchers to solve or to address some contemporary problems facing the society. Summary of the importance of inaugural lecture can be found in the following excerpt from Late Professor R. O. Ayeni’s inaugural lecture at Ladoke Akintola University of Technology of Thursday, 20th September, 2012:

> *“The lecturer is expected to present an overview of his/her research and update colleagues on the current and future plans, and introduce the research to a wider audience (Imperial College, London, 2012.Furthermore, it also provides opportunity for the Department to engage with broader audiences inside and outside the institution to*

*establish new collaborations, strengthen the existing relationship and catch up with Alumni. The University can also use inaugural lecture to engage with audiences with a broader interest research, including fund raiser and decision makers from government, academic and industry (Imperial College, London, 2012)".*

Valedictory lecture is usually given at the end of retirement .

## 3. TITLE OF THE LECTURE

I would have titled this inaugural lecture "From grass to grace" in order to reveal the work of God in my life. Without His Grace I cannot stand before you today to present inaugural lecture. However since He has made me a Professor of Mathematics I decided to caption it **"MATHEMATICAL FIXATION ALGORITHM FOR CORRUPTION"**. The title was carefully chosen in order to make some valuable revelations that will fix some Nigeria Problems especially corrupt practices. Please don't sleep I want you to listen attentively and enjoy the lecture. At the end of this lecture, you will be completely transformed; your life will no longer be the same; your attitude towards Mathematics will positive and the phobia for mathematics will disappear from ever.

## 4. MATHEMATICS DEFINITION

Before given you the definition of mathematics, I will give you the following quote on mathematics:

> "*Education should be started with mathematics for it forms well designed brain that are able to reason right. It is even admitted that those who have studied mathematics during their childhood should be trusted, for they have acquired solid bases for arguing which become to them a sort of second nature" (IbnKhaldun, al-maqaddima, born in 1332,Tuni,historian, Sociologist, Philosopher ..)*

Britannica concise encyclopedia defines mathematics as "science of structure, order, and relation that has evolved from counting and describing the shapes of object. It deals with logical reasoning and quantitative calculations". The above definition is synonymous with "things which can be counted. No wonder the Pythagos and Pythagoreans said that number rules the universe. They postulated that binary (0, 1) system is the basis of this universe. That is things are created in pairs (i.e Man/woman, Darkness/light, on/off etc). Just think of what will happen if mathematics does not exist.

## 5. CLASSIFICATIONS OF MATHEMATICS

Mathematics is usually classified in to two major groups. Viz-a-Viz **Pure** and **Applied**. However, because of the advent of Digital computer, **Computational** mathematics can be adde.

**Pure Mathematics** is the area of mathematics that is heavily abstract. The application might not be immediately known. It consists of subject such as Complex Analysis, Topology, Number Theory, Abstract Algebra etc. Its application can be found in the areas of computer science, Telecommunication etc. Numbers 0 and 1 are used for computer architecture. Without numbers 0 and 1 there will be no computer science.

**Applied Mathematics** is the aspect of mathematics that has immediate application in our daily operations or activities. The subjects included in applied mathematics are Optimization, Numerical Analysis, Differential Equations, Statistics and Dynamics etc.

**Computational** Mathematics has computer on the focal point of its operations. Numerical Analysis and Applied functional Analysis are examples of **Computational** mathematics.

Classification of Mathematics is not mutually exclusive.

## 6. PRACTICAL OR UTILIZATION VALUES OF MATHEMATICS

Seven areas of mathematical values in the civil society have been identified as follows:

i. Practical and Utilitarian Values
ii. Disciplinary Values
iii. Cultural Values
iv. Social Values
v. Moral Values
vi. Aesthetic Values and
vii. Recreation Values

Practical and Utilitarian Value will be considered here.

Mathematics is used every day by everybody in various aspect or disciplines. Mathematics is with you everywhere anywhere. Without mathematics life will be meaningless. Basically, real life application of mathematics can be found in literatures and internet. Some of the applications include the following:

- Tailoring (measurement and designs etc)
- Communications (Phone recharge etc)
- Medicines and Pharmacy
- Carpentry
- Financial Institutions
- Geometry ( interior decoration, tiling etc)
- Cooking

- Architectural designs (telecommunication mast etc0
- Banking sector (interest rate, credit etc)
- Games (ludo, Ayo etc)
- Geometry in art (Toyota car logo etc)
- Lottery: Probability
- Commerce (sale promotion etc)
- Carrier opportunity: Most of the courses in the Universities requires at least 'o' level mathematics. Without mathematics a student may not be able to realize his/her future carrier ambition.
- 0 and 1: The language of digital computer. It is used for computer architecture. The advent of digital computer has changed the direction of science and technology for the development of mankind.

Just think about what will happen if there is no mathematics at all. We must be grateful to the God Almighty for giving us mathematics by saying 'Thank you God for giving us mathematics' after our daily devotional prayers.

## 7. MY RESEARCH ACTIVITIES

Madam Vice Chancellor Ma, I work in the area of Computational Mathematics and my specific area of specialization is Optimization (Control Theory and Application). I have been working on four major topics in this area. The topics are as follows:

i. Conjugate Gradient (CGM) and Extended Conjugate Gradient (ECGM) Methods;
ii. Hardy's Inequality (integral type);
iii. Pattern recognition and
iv. Fuzzy Logic.

I will briefly describe each of the work one after the other and some areas of application to Nigeria's problem will be identified

### a. Conjugate Gradient (CGM) and Extended Conjugate Gradient (ECGM) Methods

We begin by considering descent with a functional $f$ on a Hilbert space $H$ in which $f$ is a Taylor series expansion truncated after the second order terms, namely:

$$f(x) = f_0 + < a, x >_H + \frac{1}{2} < x, Ax >_H \qquad (1)$$

Where $A$ is an$nxn$ symmetric positive definite matrix operator on the Hilbert space $H$. $\alpha$ is a vector in $H$ and $f_0$ is a constant term.

Let us also consider what is termed conjugate descent with $f$. With conjugate descent, it is assumed that a sequence

$$\{p_i\} = p_0, p_1, \dots, p_k, ..$$

is available with the members of the sequence conjugate with respect to the positive definite linear operator $A$.

By conjugate with respect to $A$ we mean that

$$< p_i, Ap_i >_H = \begin{cases} \neq 0, & if\ i \neq j \\ = 0, & if\ i = j \end{cases}$$

In the case here, $A$ is assumed positive definite so $< p_i, Ap_i >_H > 0$.

**Steps involved in conventional conjugate gradient method algorithm (CGM)**

With conjugate gradient descent, as with any descent method.The steps involved in CGM algorithm is as follows:

**Step 1**

Simply involves guessing the first sequence$x_0$. The remaining members of the sequence are then calculated as follows:

**Step 2**

$$p_0 = -g_0 = -(a + Ax_0)$$

($p_0$is the descent direction and $g_0$ is the gradient of $F(x)$ when $x = x_0$)

**Step3**

$$x_{i+1} = x_i + \alpha_i p_i, \qquad \alpha_i =< g_i, g_i >_H / < p_{i,} Ap_i >_H$$

$g_{i+1} = g_i + a_i Ap_i$;

$\alpha$ is the step length

$p_{i+1} = -g_{i+1} + \beta_i p_i$; $\beta_i =< g_{i+1}, g_{i+1} >_H / < g_i, g_i >_H$

**Step 4**

if$g_{i=0}$ for some $i$ terminate the sequence else, set $i = i + 1$ and go to step 3.

If $H = R^n$, operator $A$ turns out to be a positive definite symmetric matrix operator and for this case can easily be computed. The CGM algorithm has a well worked out theory with an elegant convergence profile. It has been proved that the algorithm converges, at most, in n iterations in a well posed problem and the convergence rate is given as:

$$E(x_n) \le \left(\frac{1-\frac{m}{M}}{1+\frac{m}{M}}\right)^{2n} E(x_0)$$

Where m and M are smallest and spectrums of matrix A respectively.

That is, for an n dimensional problem, the algorithm will converge in at most n iterations.

This Conventional CGM algorithm, due to Hestene and Stiefel, was originally designed for the minimization of a quadratic objective functional.

CGM appears to be the most popular among the descent iterative methods because of its simplicity, elegance and the convergence property in handling quadratic functional. With the above qualities in mind, Ibiejugba and his associates [ 1] worked on the possibility of applying CGM algorithm to continuous optimal cost functional. Their own version of CGM is called the Extended Conjugate Gradients Method.

We know that if $H = \mathbb{R}^n$ the operator $A$ turns out to be a positive definite symmetric constant matrix and A can easily be computed in quadratic functional so that CGM Algorithm will be appropriate for the solution but if $H \neq \mathbb{R}^n$ the situation becomes very difficult. This is what motivated Ibiejugba [1], knowing explicitly the control operator $A$ so as to compute $AP_i$ needed for the step length $\alpha_i$. Ibiejugba [1] successfully constructed this control operator $A$. He (Ibiejugba) then used the constructed operator to formulate his own method called the Extended Conjugate Gradient Method (ECGM). The formalism of CGM was adopted in the construction of the Extended Conjugate Gradient Method (EGCM) algorithm. Though the construction of the operator is difficult, the joy of it is that no approximation is involved. The concept of functional analysis was extensively used in the construction. It was originally formulated to solve problems in the following class:

Minimize $\int_0^{\delta}\{x^T(t)px(t) + u^T(t)qu(t)\}dt$ (2)

Subject to

$$\dot{x}(t) = cx(t) + du(t)$$

$0 \le t \le \delta(\delta; \text{given})$ ; $x^T(t)$ denotes the transpose of $x(t)$ ,$\dot{x}(t)$ stands for the first derivative of $x(t)$ with respect to $t$ .$x(t)$is the nx1 – state vector, $u(t)$ is the qx1 – control vector applied to the system at time $t$. c and d are nxn, nxq constant matrices respectively, while $p$ and $q$ are symmetric , positive definite , constant square matrices of dimensions $n$ and $q$ respectively. The control operator $A$ associated with [1] satisfies

$\langle Z, Z\rangle_K = J(x, u, \mu) = \int_o^\delta\{x^T(t)Px(t) + u^T(t)qu(t)dt\} + \mu\int_o^\delta\{\|\dot{x}(t) - cx(t) - du(t)\|^2\}dt$
(3)

By transforming (2) into an unconstrained optimal control problem, $\mu$ is a penalty constant greater than zero, $K$ is defined by $K = H_1[0, \delta]xL_2^q[0, \delta]$ Where $H_1[0, \delta]$ denotes Sobolev space of the absolutely continuous functions $x(\cdot)$ square integrable over the closed interval$[0, \delta]$. $L_2^q[0, \delta]$stands for the Hilbert space consisting of the equivalence classes of square integrable functions from $[0, \delta]$ into $R^2$, with norm denoted by $\|\cdot\|_E$ defined by $\|u\| = \left\{\int_o^\delta\|u(t)\|^2\,dt\right\}^{\frac{1}{2}}$ and with scalar product conventionally denoted by $\langle . , . \rangle$ and defined by $\langle u_1, u_2\rangle = \int_o^\delta\langle u_1(t), u_2(t)\rangle_E dt$ . Furthermore, $\|\cdot\|_E$ and $\langle . , . \rangle_E$ denote the norm and scalar product in Euclidean $q$-dimensional space. The result of the constructed operator $A$ is as follows:

$$AZ(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_{11}x(t) + & A_{12}u(t) \\ A_{21}x(t) + & A_{22}x(t) \end{bmatrix}, \quad (4)$$

Where $A_{11}$ is such that

$$\begin{aligned}(A_{11}x)(t) = -\mu(\dot{x}(0) - cx(0))sinht + \mu\int_0^t(\dot{x} - cx)(s)cosh(t-s)ds \\ - \int_0^t[(q + \mu c^T c)x - \mu cx](s)\,sinh(t-s)ds \\ + \left\{-\mu sinhT(x(0) + cx(0)] + \mu\int_0^T(x \dot{-} cx)(s)cosh(T-s)ds\right. \\ \left. - \int_0^T[(q + \mu c^T c)x - \mu c\dot{x}](s)sinh(T-s)ds\right\}exp(T), \qquad 0 \le t \le T\end{aligned}$$

$$(A_{12}u)(t) = cu(t) + \mu d^T du(t), \qquad 0 \le t \le T$$

$$(A_{21}x)(t) = \mu c^T dx(t) + \mu du(t), \qquad 0 \le t \le T$$

$$\begin{aligned}(A_{22}d)(t) = \mu du(s)Sinht - \mu\int_o^t du(s)Cosh(t-s)ds + \int_o^T c^T\,dSinh(t-s)ds + \{\mu du(0)Sinht \\ - \int_o^T \mu du(S)Cosh(t-s)ds + \mu\int_o^T du(S)Sinh(t-s)ds\}exp(T) \quad 0 \le t \le T\end{aligned}$$

The desired $AP_i$ can now be applied when calculating $g_{i+1} = g_i + \alpha_i AP_i$ in the CGM algorithm which will now allow us to exploit the simplicity of the CGM algorithm. Therefore the ECGM algorithm is given as follows:

**Step 1**

It involves guessing the first sequence $x_0$. The remaining members of the sequence are then calculated as follows:

**Step 2**

$p_0 = -g_0$

($p_0$is the descent direction and $g_0$ is the gradient of the cost functional when $x = x_0$)

**Step3**

$$x_{i+1} = x_i + \alpha_i P_{x,i}$$
$$u_{i+1} = u_i + \alpha_i P_{u,i}$$
$$g_{x,i+1} = g_{x,i} + \alpha_i AP_{x,i}$$
$$g_{u,i+1} = g_{u,i} + \alpha_i AP_{u,i}$$
$$P_{x,i+1} = -g_{x,i+1} + \beta_i P_{x,i}$$
$$P_{u,i+1} = -g_{u,i+1} + \beta_i P_{u,i}, \quad \text{where}$$
$$\alpha_i = \frac{\langle g_i, g_i \rangle}{\langle P_i, AP_i \rangle} \text{ and } \beta_i = \frac{\langle g_{i+1}, g_{i+1} \rangle}{\langle g_i, g_i \rangle}$$
$$\begin{aligned} AP_i = \Bigg( & -\mu \sinh t\left(J_{x,0} - cp_{x,0}\right) + \mu \int_0^t (cj_i - cp_{x,i}) \cosh(t-s)\, ds - \int_0^t [q + \mu c^T c] P_{x,i} \\ & - \mu c J_{x,i} \sinh(t-s)\, ds \\ & + \Big\{ \mu \sinh \sigma \left[-j_{x,0} - cp_{x,0}\right] \\ & + \mu \int_0^T \left(J - cp_{x,i}\right) \cosh(\sigma - s)\, ds \\ & - \int_0^t [q + \mu c^T c] p_{x,i} - \mu c j_{x,i}] \sinh(\delta - s) ds \Big\} \exp(\sigma) + cu_i(t) \\ & + \mu d^T du(t);\ \mu\left[c^T dp_{x,i} - dJ_{x,i}\right] \\ & + \mu \Big\{ Du(0) \text{Sinh} t - \int_o^t Du_i\,(s) \cosh(\delta - s)\, ds \\ & + \mu \int_O^\delta c^T\, Du_i(s) \text{Sinh}(\sigma - s) ds \Big\} \exp(\delta) \Bigg), \end{aligned}$$

and where we have used the following notations:

$J_i = J(x_i, u_i, \mu)$

$J_{x,i} = J_x(x_i, u_i, \mu)$

$J_{u,i} = J_u(x_i, u_i, \mu)$

$p_{x,i} = p_x(x_i, u_i, \mu)$

$p_{u,i} = p_u(x_i, u_i, \mu)$

$p_{u,i} = p_u(x_i, u_i, \mu)$

$p_x(x_i, u_i, \mu) = \int_o^t J_x\,(x_i(s), u_i(s), \mu)ds$

and

$p_u(x_i, u_i, \mu) = \int_o^t J_u\,(x_i(s), u_i(s), \mu)ds$

**Step 4**

If $g_i$ satisfies the tolerance, for some $i$ terminate the sequence else, set $i = i + 1$ and go to step 3.

To the best of my knowledge I was the first researcher to implement ECGM algorithm numerically (confirmed by my P.hD. supervisor, Prof. M. A. Ibiejugba).

Basically the ECGM algorithm was formulated by Ibiejugba et al to solve problems in quadratic cost functionals of the type

Minimize $\int_0^\delta \{ax^2(t) + bu^2(t)\}dt$

Subject to (5)

$\dot{x}(t) = cx(t) + du(t)$

However I extended the algorithm to the solution of Reaction Diffusion problem.

**Reaction Diffusion System**

Reaction Diffusion Systems are of great importance to Engineers, Scientists and Computational Mathematicians. Many researchers have worked on Coupled Linear Reaction Diffusion Equations but there are no explicit solutions. Hill [2] outlined a general procedure for obtaining a closed form representations of the solutions $u(x,t)$ and $v(x,t)$ for the linear reaction diffusion equation:

$$\frac{\partial u}{\partial t} = D_1\nabla^2 u - au + bv$$

$$\frac{\partial v}{\partial t} = D_2\nabla^2 v + cu - dv\,, \qquad (5.1)$$

where $D_1, D_2, a, b, c$ and $d$ are all nonnegative constants.

He showed that closed form solutions of (1.1) can be given in terms of arbitrary integral heat functions $h_1(x,t)$and $h_2(x,t)$.

That is functions that satisfy classical heat equation

$$\frac{\partial h}{\partial t} = \nabla h. \tag{5.2}$$

and in particular; he established that the formal solutions of (1.1) are

$$u(x,t) = e^{-at} h_1(x, D_1 t) + \frac{b^{1/2} e^{-\lambda At}}{(D_1 - D_2)} \int_{D_2 t}^{D_1 t} e^{-\mu\xi} \left\{ e^{1/2} \frac{(\xi - D_2 t)^{1/2}}{D_1 t - \xi} I_1(n) h_1(x,\xi) + b^{\frac{1}{2}} I_0(n) h_2(x,\xi) \right\} d\xi \tag{5.3}$$

$$v(x,t) = e^{-at} h_2(x, D_2 t) + \frac{c^{1/2} e^{-\lambda At}}{(D_1 - D_2)} \int_{D_2 t}^{D_1 t} e^{-\mu\xi} \left\{ e^{1/2} \frac{(D_1 t - \xi)^{1/2}}{\xi - D_2 t} I_2(\tau) h_2(x,\xi) + c^{\frac{1}{2}} I_0(\tau) h_1(x,\xi) \right\} d\xi, \tag{5.4}$$

where the constants $\lambda$ and $\mu$ are given by the following equations:

$$\lambda = \frac{(aD_2 - dD_1)}{D_1 - D_2} \tag{5.5}$$

$$\mu = \frac{(a-d)}{D_1 - D_2} \tag{5.6}$$

$I_0, I_1, I_2$are the usual modified Bessel functions and $\tau$ is given by

$$\tau = \frac{2(bc)^{1/2}}{D_1 - D_2} [(D_1 t - \xi)(\xi - D_2 t)]^{\frac{1}{2}} \tag{5.7}$$

Hill [2] considered the application of these general formulae to the stability problems arising from a model of an arms race which incorporates the features of deteriorating armaments.

This situation is as follows:

Richardson as reported in [5] proposed that the military spending of two nations locked in an arms race can be modeled by the following linear systems:

$$\frac{dp}{dt}(t) = -ap(t) + bq(t) + g \tag{5.8}$$

$$\frac{dq}{dt}(t) = cp(t) - dq(t) + h \tag{5.9}$$

Where $p(t)$ and $q(t)$ denote armament levels of the two nations at time *t* and *a, b, c, d, g* and *h* denote positive constants. The constants $b$ and $c$ are called "*Threat Coefficients"* and they signify the degree to which a nation is stimulated by another nation's weapon stock to increase her own stocks. The constants $a$ and d, called "*Fatigue Coefficients"*, are measures of prevailing economic circumstances which inhibit armament build-up. The constants $g$ and $h$denote measures of the circumstances which prevent a complete disarmament in the situation when both nations have zero armaments. A "Balance of Power" situation results when the armament levels remain constant over a long period of time and these levels are given [ 2 ] by the following equation

$$p_0 = \frac{gd+hb}{(ad-bc)} \tag{5.10}$$

$$q_0 = \frac{gc+ha}{(ad-bc)} \tag{5.11}$$

$$(ad - bc) > 0$$

Gopalsamy as reported in [1] developed Richardson model and proposed that the armament levels $p(x,t)$ and $q(x,t)$ satisfy

$$\frac{\partial p}{\partial t} + e_1 \frac{\partial p}{\partial x} = \frac{a_1^2}{2}\frac{\partial^2 p}{\partial x^2} - ap + bq + g \tag{5.12}$$

and

$$\frac{\partial q}{\partial t} + e_2 \frac{\partial p}{\partial x} = \frac{a_2^2}{2}\frac{\partial^2 p}{\partial x^2} + cq - dp + h(5.13)$$

where$e_1, e_2, a_1$ *and* $a_2$ denote positive constants that are also previously defined.

Hill [2] further developed a model and asserted that in order to investigate the stability of power situation $(p_0, q_0)$ given by (5.10) and (5.11) he sets

$$p(x,t) = p_0 + u(x,t) \tag{5.14}$$

and

$$q(x,t) = q_0 + v(x,t) \tag{5.15}$$

so that from (5.12) and (5.13), we have that:

$$\frac{\partial u}{\partial t} = D_1 \frac{\partial^2 u}{\partial x^2} - e_1 \frac{\partial u}{\partial x} + au - bv$$

$$0 \le t, x \le 1$$

$$\frac{\partial v}{\partial t} = D_2 \frac{\partial^2 v}{\partial x^2} - e_2 \frac{\partial v}{\partial x} + cu - dv$$

$D_i = \frac{\alpha_i^2}{2}(i = 1,2)$ (5.16)

$u(x,0) = 0, v(x,0) = 0, u(0,t) = u_0, v(0,t) = v_0$

$u(x,t), v(x,t) \to 0\ as\ x \to \infty$.

Despite the large amount of qualitative theory for reaction diffusion equations that have been carried out, the application and interpretation of the results are difficult.

Because of the above short comings, we are concerned with the transformation of linear reaction diffusion equations into a control problem so that the application of optimization techniques can be employed for the solution of the resulting problem. Basically, this work considers the transformation of Coupled Linear Reaction Diffusion Equations into optimization so that some numerical methods can be used for the solution of resulting problem. This will make possible the utilization of known and unknown features of reaction diffusion equations to be used to advance theorems and propositions in Mathematical Sciences and Computation.

We shall now state and prove an important theorem in this work.

1. **Theorem:**

The reaction diffusion problem (5.1) to (5.16) is equivalent to the following control problem:

$Minimize\ \int_0^T \{au^2(t) + av^2(t)\}\, dt$ (6)

Subject to

$\dot{u}(t) - \dot{v}(t) = cv(t) + du(t).$

**Proof:**

We shall now transform the whole reaction diffusion system (5.1) to (5.16) into a control problem in the following manner.

Minimize $\int_0^1 \int_0^1 [u^2(x,t) + v^2(x,t)]\, dxdt$

Subject to

$$\frac{\partial u}{\partial t} - D_1 \frac{\partial^2 u}{\partial x^2} + e_1 \frac{\partial u}{\partial x} - au + bv = 0$$

$\frac{\partial v}{\partial t} - D_2 \frac{\partial^2 v}{\partial x^2} + e_2 \frac{\partial v}{\partial x} - cu + dv = 0$(6.1)

$0 \leq x, t \leq 1,$

with the following initial and boundary conditions:

$$u(x,0) = 0, v(x,0) = 0, u(0,t) = u_0, v(0,t) = v_0$$

$u(x,t), v(x,t) \to 0 \; as \; x \to \infty.$

$\frac{\partial u(x,1)}{\partial t} = \frac{\partial v(x,1)}{\partial t} = 1..$ (6.2)

The boundary conditions at *x* equals zero represent the fact that both nations locked in the arms race are maintaining a perfect level of undeteriorated strategic weapon system and the integral given by (6.1) is a measure of military spending.

To obtain an explicit solution of the boundary value problem (6), Gopalsamy as reported in [2] assumed that $e_1 = e_2, D_1 = D_2$ and $a = d$. We also adopt these values for simplicity and consistency.

Let $v(x,t) = v(t)\frac{sin\pi x}{l}.\ 0 \leq x \leq l \equiv 1,$

$$u(x,t) = u(t)\frac{sin\pi x}{l}.\ 0 \leq x \leq l \equiv 1.$$

Thus we have

$$v_x(x,t) = \sum_{i=1}^{n} \dot{v}_\iota(t) sin\pi x$$

$$u_x(x,t) = \sum_{i=1}^{n} \dot{u}_\iota(t)\, sin\pi x$$

$$v_{xx}\,(x,t) = -\pi^2 \sum_{i=1}^{n} v_i\, sin\pi x$$

$$u_{xx}\,(x,t) = -\pi^2 \sum_{i=1}^{n} u_i\, sin\pi x$$

$$v^2(x,t) = \sum_{i=1}^{n} v_i^2(t)\, sin^2\pi x.$$

$$u^2(x,t) = \sum_{i=1}^{n} u_i^2(t)\, sin^2 \pi x.$$

And on substituting the values of $v^2(x,t),\ u^2(x,t)$ in the integral (6.1), we obtain:

$\int_0^1 \int_0^1 [v^2(x,t) + u^2(x,t)]\, dxdt = \int_0^1 \int_0^1 [\sum_{i=1}^{n} v_i^2(t)\, sin^2\pi x + \sum_{i=1}^{n} u_i^2(t)\, sin^2\pi x]\, dxdt.$

$= \frac{1}{2}\int_0^1 \int_0^1 \left[\sum_{i=1}^{n} v_i^2(t)\,[1 - cos2\pi x] + \sum_{i=1}^{n} u_i^2(t)\,[1 - cos2\pi x]\right] dxdt.$ (6.3)

$$= \frac{1}{2}\int_0^1 \left\{\sum_{i=1}^{n} v_i^2(t)\left[x - \frac{\sin 2\pi x}{2\pi}\right]_0^1 + \sum_{i=1}^{n} u_i^2(t)\left[x - \frac{\sin 2\,\pi x}{2\pi}\right]_0^1\right\} dt.$$

$$= \frac{1}{2}\int_0^1 \left\{\sum_{i=1}^{n} v_i^2(t)\,[1 - 0 - (+0 - 0)] + \sum_{i=1}^{n} u_i^2(t)\,[1 - 0 - (+0 - 0)]\right\} dt.$$

$$= \frac{1}{2}\int_0^1 \left\{\sum_{i=1}^{n} v_i^2(t) + \sum_{i=1}^{n} u_i^2(t)\right\} dt.$$

Since it is a minimization problem the $\frac{1}{2}$ in the right hand side can be omitted.

Thus we have

$\int_0^1 \int_0^1 [v^2(x,t) + u^2(x,t)]\, dxdt.$

$$= \int_0^1 \left\{\sum_{i=1}^{n} v_i^2(t) + \sum_{i=1}^{n} u_i^2(t)\right\} dt$$

Following the idea of Gopalsamy as reported in [1]

$$e_1 = e_2 = 1. \qquad (6.4)$$

Equating the constraints in (2.1), we obtain

$$\frac{\partial u}{\partial t} - D_1 \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + au - bv = \frac{\partial v}{\partial t} - D_2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial v}{\partial x} - cu + dv.$$

On substituting values for $u_i, v_i, u_{xx}, v_{xx}, u, v$ in the last equation, we obtain

$$\sum_{i=1}^{n} \dot{u}_i(t) sin\pi x - D_1\pi^2 \sum_{i=1}^{n} \mathrm{u_i} \sin \pi x + a \sum_{i=1}^{n} u_i(t) \sin \pi x - b \sum_{i=1}^{n} v_i(t) \sin \pi x$$
$$= \sum_{i=1}^{n} \dot{v}_i(t) sin\pi x$$
$$- D_2\pi^2 \sum_{i=1}^{n} \mathrm{v_i} \sin \pi x - c \sum_{i=1}^{n} u_i(t) \sin \pi x + d \sum_{i=1}^{n} v_i(t) \sin \pi x$$

Dividing both sides by $sin\ \pi x$, since Sin $\pi x \neq 0,\ \forall\ 0 < x < 1$ we obtain

$$\sum_{i=1}^{n} \dot{u}_i(t) + D_2^2\pi^2 \sum_{i=1}^{n} \mathrm{u_i} + a \sum_{i=1}^{n} u_i(t) - b \sum_{i=1}^{n} v_i(t).$$
$$= \sum_{i=1}^{n} \dot{v}_i(t) + D_2^2\pi^2 \sum_{i=1}^{n} \mathrm{v_i} - c \sum_{i=1}^{n} u_i(t) + d \sum_{i=1}^{n} v_i(t).$$

Rearranging and dropping the summation sign, we obtain

$$\dot{u}_1(t) - \dot{v}_1(t) = [D_2\pi^2 + d + b]v_1(t) + [-D_1\pi^2 - a - c]u_1(t)$$
$$+ \dot{u}_2(t) - \dot{v}_2(t) = [D_2\pi^2 + d + b]v_2(t) + [-D_1\pi^2 - a - c]u_2(t)$$
$$+ \dots +$$
$$\dot{u}_n(t) - \dot{v}_n(t) = [D_2\pi^2 + d + b]v_n(t) + [-D_1\pi^2 - a - c]u_n(t). \quad (6.5)$$

The above can be put in a compact form in the following manner:

$$\dot{u}_i(t) - \dot{v}_i(t) = Cv_i(t) + Du_i(t). \quad (6.6)$$

Where

$$C = D_2\pi^2 + d + b, \qquad i = 1,2, \dots, n.$$
$$D = -D_1\pi^2 - a - c, \qquad i = 1,2, \dots, n. \quad (6.7)$$

Consequently, problem (2.1) reduces to

Miniimize $\int_0^T \{\sum_{i=1}^n v_i^2(t) + \sum_{i=1}^n u_i^2(t)\} dt$

Subject to

$\dot{u}_i(t) - \dot{v}_i(t) = Cv_i(t) + Du_i(t)$. (6.8)

Where $i = 1, .., n.$

$C = D_2\pi^2 + d + b,$

$D = -D_1\pi^2 - a - c,$

This can be put as a one dimensional problem in the form:

$$Minimize \int_0^T \{au^2(t) + bv^2(t)\}\, dt.$$

Subject to

$$\dot{u}(t) - \dot{v}(t) = cv(t) + du(t). \tag{6.9}$$

This completes the proof of the theorem.

I am the owner (author) of equation (6.9)

This last problem is designated system of reaction diffusion control problem equations. The equation is unique by the fact that both state and control vectors in the left hand of the constraint are of derivative type.

This problem has enabled me and my former Ph.D. student now Dr. AminuLukuman to model nuclear safety as a cost functional with dynamic constraint which is reported in his Ph/D. thesis and in our publications. For details see[80].

b. **Hardy's Inequality (integral type)**

The following famous classical inequality was proved in 1920 by G. H. Hardy (see, [78]): If $1 < p < \infty$, $A_n = \sum_{k=1}^n a_k$ and $a_n = \{a_k\}$ is a sequence of non-negative real

numbers, then

$$\sum_{n=1}^{\infty} | \frac{1}{n} A_n |^p \leq C_p \sum_{n=1}^{\infty} | a_n |^p \tag{1.1}$$

and in 1925 he proved the continuous counterpart:

**Theorem 1.1.** *Let* $f(x)$ *be a non-negative p-integrable function defined on* $(0,\infty)$, *and* $p>1$. *Then,* $f$ *is integrable over the interval* $(0,x)$ *for each* $x$ *and the following inequality:*

$$(1.2) \qquad \int_0^\infty \left[\frac{1}{x}\left(\int_0^x f(y)dy\right)\right]^p dx \le \left(\frac{p}{p-1}\right)^p \int_0^\infty f(x)^p\, dx$$

*holds,* where $C_p = \left(\frac{p}{p-1}\right)^p$ *is the best possible constant*.

This inequality was developed in his attempt to provide an elementary proof to the following famous Albert Hilbert double series theorem [78]:

**Theorem 1.2.** *If* $\sum_{m=1}^\infty a_m^2 < \infty$ *and* $\sum_{n=1}^\infty b_n^2 < \infty$, *where* $a_m \ge 0$ *and* $b_n \ge 0$, *then the double series:* $\sum_{n=1}^\infty \sum_{m=1}^\infty \frac{a_m b_n}{m+n}$ *converges. In particular,*

$$(1.3) \qquad \sum_{n=1}^\infty \sum_{m=1}^\infty \frac{a_m b_n}{m+n} \le \pi \left(\sum_{m=1}^\infty a_m^2\right)^{\frac{1}{2}} \left(\sum_{n=1}^\infty b_m^2\right)^{\frac{1}{2}}$$

In his attempt to simplify this theorem, he needed an estimate for arithmetic means of the form:

$$\sum_{n=1}^\infty \left|\frac{1}{n} A_n\right|^2 \le C_2 \sum_{n=1}^\infty |a_n|^2$$

with both $a_n$ and $A_n$ are as defined above. That was what lead him to inequality (1.2).

In 1928, Hardy[16] obtained a generalized form of (1.2), namely that if $p \ge 1$ and $k \ne 1$,

Then

$$(1.4) \qquad \int_0^\infty x^{-k}\left(\int_0^x f(t)dt\right)^p dx \le \left(\frac{p}{k-1}\right)^p \int_0^\infty x^{p-k} f(x)^p dx \,(p \ge 1, k > 1)$$

and also the dual form of this inequality

$$(1.5) \qquad \int_0^\infty x^{-k}\left(\int_0^x f(t)dt\right)^p dx \le \left(\frac{p}{1-k}\right)^p \int_0^\infty x^{p-k} f(x)^p dx \,(p \ge 1, k > 1).$$

The constant $(\frac{p}{|k-1|})^p$ is the best possible in both case.

Furthermore, Hardypointed out that if $k$ and $f$ satisfy the conditions of the above results, then (1.4) and (1.5) hold in the reversed direction with $0 < p \leq 1$.

Thereafter, inequality (1.2) was extended and generalized in many directions, for example, if

$$T : L^p(\mathrm{R}) \to L^p(\mathrm{R})$$

where T is an integral operator of the form:

$$(Tf)(x) = \int_{-\infty}^{x} K(x,y) f(y) dy$$

or

$$(T^* f)(x) = \int_{x}^{\infty} K(y,x) f(y) dy$$

then, Hardy's inequality is expressible in the operator form as

(1.6) $\int_0^\infty (Tf)(x)^p dx \leq A(K,p) \int_0^\infty f(x)^p dx$

*where* $A(K,p)$ *is a constant independent of* $f$, $p > 1$ *and* $K(x,y) = \frac{1}{x}$ *if* $y \leq x$ *and* 0 *otherwise.*

In the early seventies, a new dimension was introduced into inequality (1.2) and emphasis was later shifted to finding the necessary and sufficient conditions on the non-negative weight functions $\omega$ and $\nu$ such that, the norm inequality:

(1.7) $\| \omega Tf \|_p^p \leq A(K.p) \| f\nu \|_p^p$

is valid, where $p > 1$, $f$ is a non-negative function defined on $[0, x]$,$A(k,p)$ is a constant depending on $k$ and $p$ but independent of the functions $f$and $\grave{A}(x,y) = \frac{1}{x}$ and if $y \leq x$ and 0 otherwise.

Many researchers investigated independently the necessary and sufficient conditions on the non-negative weight functions and which ensure that the inequality:

(1.8) $(\int_0^\infty (\omega(x) \int_0^x f(t)dt)^p dx)^{\frac{1}{p}} \leq C(\int_0^\infty (f(x)\nu(x))^p dx)^{\frac{1}{p}}$

holds, where $f$ and $C = A(K,p)$ is as defined in (1.7). It can be readily observed that (1.8) reduced to (1.2) with $w(x) = x^{-1}$ and $v(x) = 1$

Muchenhoupt [26] studied inequality (1.6) and gave conditions on the non-negative weight functions $w(x)$ and $v(x)$ such that (1.7) is valid. He raised the question that given the weight function $w(x)$, under what condition will there exist a weight function$v(x)$, such that

(1.9) $$\int_X (\mathrm{Tf})(\mathrm{x})^{\mathrm{p}} d\mu \leq \int_0^x \mathrm{f}(\mathrm{x})^{\mathrm{p}} \nu d\mu$$

holds for all $f \geq 0$.

In their attempt to simplify this problem a partial solution to this question was provided and two new open problems were posed. That is, the characterization of weights $\omega$ for which for which there exist $\nu < \infty$ $\mu$-almost everywhere such that (1.9) holds, where $T$ is a sublinear operator and secondly, for $1 < p, q < \infty$, those weight functions $\omega$ and $\nu$ are to be characterized when $T$ maps $L^p(\nu)$ to $L^q(\omega)$ such that:

(1.10) $$\left(\int_X (\mathrm{Tf})(\mathrm{x})^{\mathrm{q}} \omega d\mu\right)^{\frac{1}{\mathrm{q}}} \leq \left(\int_0^x \mathrm{f}(\mathrm{x})^{\mathrm{p}} \nu d\mu\right)^{\frac{1}{\mathrm{p}}}$$

holds for all $f \geq 0$, $\nu < \infty$, and for every $\mu$-almost everywhere on $X$. Inequality (1.1) has been treated partially when $T$ is the Hardy-Littlewood maximal function; while (1.2) has been treated partially for the case of fractional integrals.

Rauf and Imoru[78] provided partial solution to the open problems when $T$ is a sublinear operator while Rauf and Omolehin[78] provided partial solution to the same problems in the case in which $T$ is a non-linear integral operator.

Some works have shown that Hardy's inequality with mixed norms in a generalized form as:

$$\left(\int_0^\infty (\omega(x)\int_0^x f(t)dt)^q \, dx\right)^{\frac{1}{q}} \leq C\left(\int_0^\infty (f(x)\nu(x))^p \, dx\right)^{\frac{1}{p}}$$

holds for non-negative weight function $f$ defined on $[0,\infty]$ if and only if

$$\sup_{r>0}\left(\int_0^\infty \omega(x)^q \, dx\right)^{\frac{1}{q}}\left(\int_0^r \nu(x)^{-p'} \, dx\right)^{\frac{1}{p'}} \equiv K < \infty,$$

where $1 \leq p \leq q \leq \infty$, $\omega(x)$ and $\nu(x)$ are non-negative weight functions, $p$ and $p'$ are conjugate exponents, and $K$ is a positive constant independent of $f$.

Also weighted case for negative powers has been considered and the result extended to the case where $p, q < 0$ and $0 < p, q < 1$. The reverse Hardy inequality wasvconsidered:

$$(1.11) \qquad \left(\int_0^\infty (f(x)v(x))^p\,dx\right)^{\frac{1}{p}} \leq \mathrm{C}\left(\int_0^\infty (u(x)\int_0^x f(t)dt)^q\,dx\right)^{\frac{1}{q}}$$

*The dual version of (1.2) with necessary and sufficient conditions for the validity of the inequality were also considered.*

*Conditions were given on the non-negative weight functions* $\omega(x)$ *and* $v(x)$ *which ensure that the inequality of the form:*

$$(1.12) \qquad \left(\int_{-\infty}^\infty (Tf)(x)\omega(x))^q\,dx\right)^{\frac{1}{q}} \leq C\left(\int_{-\infty}^\infty (f(x)v(x))^p\,dx\right)^{\frac{1}{p}}$$

*holds, where* $T$ *is an integral operator,* $f$ *a non-negative function,* $p$ *and* $q$ *are as defined above. Inequality (1.12) extended some of the earlier as well as recent extensions on classical Hardy's inequality (see, [77). If* $K(x, y) \equiv 1$ *and* $p = q$*, the inequality (1.12) yields (1.8) from which (1.2) can be obtained.*

*The last result was generalized to* $N$ *-dimensional Hardy's inequality:*

$$(1.13) \qquad (\int_\Omega |\mathrm{f(x)}|^{\mathrm{q}} \omega(x)d(x))^{\frac{1}{q}} \leq C(\sum_{i=1}^N \int_\Omega |\frac{\delta \mathrm{f(x)}}{\delta \mathrm{x_i}}|^{\mathrm{p}} v_i(x)dx)^{\frac{1}{p}}$$

*holds, where* $\Omega$ *is a domain in the* $N$ *-dimensional Euclidean Space* $\mathsf{R}^N$, $p$, $q$ *are positive real numbers and* $\omega, v_1, v_2, \ldots, v_N$ *are weight functions that is measurable and positive almost everywhere in* $\Omega$.

*The special case of (1.13) was investigated. By considering* $\omega(x) = v_i(x) \equiv 1$, $i = 1,2,\ldots,N$ *and have.*

$$(1.14) \qquad (\int_\Omega |\mathrm{f(x)}|^{\mathrm{q}} d(x))^{\frac{1}{q}} \leq C(\int_\Omega |\nabla \mathrm{f(x)}|^{\mathrm{p}} dx)^{\frac{1}{p}}$$

*holds, for continuous function* $f(x)$ *defined on* $(0,\infty)$, $X = (X_1, X_2, \ldots, X_N)$,

$\nabla f(x) = (\frac{\delta f(x)}{\delta x_1}, \frac{\delta f(x)}{\delta x_2}, \ldots, \frac{\delta f(x)}{\delta x_N})$, $1 < p < N$, $1 < q < \frac{N_p}{N-p}$ *and* $|\nabla f(x)|^p = \sum_{i=1}^N |\frac{\delta f(x)}{\delta x_i}|^p$ *and* $\Omega$ *is a bounded domain with Lipschitzian boundary* $\delta\Omega$ *where the admissible values of the parameter* $q$ *may change. This is called Sobolev inequality.*

*Another special case of (1.14) was considered in literature with* $p = q = 2$ *as:*

$$\int_{\Omega} |f(x)|^2\,dx \le C^2 \int_{\Omega} |\nabla f(x)|^2\,dx$$

holds. This inequality is called Friedrichs inequality.

Also, for all functions $f(x)$ whose mean value over $\Omega$ is zero:

$$\int_{\Omega} f(x)dx = 0$$

is called the Poincare inequality.

Finally, by replacing $f$ with $f^{\frac{1}{p}}$ in inequality (1.2) and letting $p \to \infty$, we have the limiting inequality

$$\int_0^{\infty} \exp\left(\frac{1}{x}\int_0^x \ln f(t)dt\right)dx \le e\int_0^{\infty} f(x)dx$$

This is called the Knopp's-Poly $a'$ inequality. For further development, remarks, extensions, generalizations and applications of inequalities (1.2), (1.3), (1.4), (1.5), (1.9), (1.10), (1.11), (1.12), (1.13) and (1.14), see for instance, [78] and the references cited therein.

This work is, therefore, consequences of Hardy-type inequalities.

The aim of our research is to determine conditions on the data of Hardy-type inequalities. These were done by introducing $n$-terms of functions for all $n \in \mathbb{N}$ on a multiple Hardy integral operator and by making one of the weight functions a power function.

## 2 Multi-Dimensional Hardy-Type Inequalities with Weights

In literature, one dimensional Hardy-type inequalities has received rigorous treatment in various directions while the multi-dimensional case has been given little attention. The characterizations for pairs of weights $(u,v)$ such that the operator $T_2 : L^p(\mathbb{R}^2_+, v) \to L^q(\mathbb{R}^2_+, u)$, is bounded in the case when $1 < p \le q < \infty$ were treated in [77], that is. If $p > 1$ then

$$\int_0^{\infty}\int_0^{\infty} |T_2 f(x,y)|^p\,dxdy \le \left(\frac{p}{p-1}\right)^{2p} \int_0^{\infty}\int_0^{\infty} |f(x,y)|^p\,dxdy$$

. However, it has recently been pointed out that the proof of classical Hardy integral inequality (1.2) will also work for the corresponding multidimensional $L^p$-spaces that is the $n$-dimensional case of classical Hardy operator:

$$\int_0^{\infty}\cdots\int_0^{\infty} |T_n f(x_1,\cdots,x_n)|^p\,dx_1\cdots dx_n \le C\int_0^{\infty}\cdots\int_0^{\infty} |f(x_1,\cdots,x_n)|^p\,dx_1\cdots dx_n$$

where

$$C = \left( \frac{p}{p-1} \right)^{np}$$

and

$$T_n f(x_1, \cdots, x_n) = \frac{1}{x_1 \cdots x_n} \int_0^{x_1} \cdots \int_0^{x_n} f(t_1, \cdots, t_n) dt_1 \cdots t_n$$

Also, the corresponding weighted mixed-norm version can be proved. The multidimensional generalized Hardy-Polya type inequality described by convex functions are discussed in this section. We use the following notation throughout the remaining sections:

$$\int_{\mathbf{t}}^{\mathbf{b}} := \int_{t_1}^{b_1} \cdots \int_{t_n}^{b_n}$$

and we have similar expression for

$$\int_0^{b_1} \cdots \int_0^{b_n}, \int_0^{x_1} \cdots \int_0^{x_n}, \int_0^{\infty} \cdots \int_0^{\infty}$$

and

$$\int_X \cdots \int_X$$

where $b_i$'s, $x_i$'s and $v_i$'s are the components of $\mathbf{b}$, $\mathbf{x}$ and $\mathbf{v}$ for all $i = 1, \ldots, n \in \mathbb{Z}_+$ respectively. All functions are measurable and $g(x) = \mathbf{x}^p$ except otherwise stated. Based We further make some new generalizations of multidimensional Hardy-type integral inequalities by introducing real function $g(x)$. Some multidimensional Hardy-type integral inequalities are obtained. Some applications are also considered. First, we give some lemmas which are fundamental to prove certain inequalities in our context.

Hardy's inequality has many applications in analysis most especially in the study of Fourier series, theory of ordinary differential equations and in providing bounds to integral operators. Due to its usefulness, this inequality has been extensively studied and generalized in various directions by a number of researchers. The implication is that inequality can never be eradicated as long as the society is dynamics..

**c. Pattern recognition**

Madam Vice Chancellor Ma, I have also worked extensively in Computer Science. Some of the members of my research group in Computer Science include Rauf, Lukuman and Enikuomehin.

Others are Drs Abikoye, Jimoh, Ameen, Babatunde and Mabayoje. We have published many papers in reputable National and International Journals. In particular, I co-supervised the Ph.D. Thesis of Dr. (Mrs) C. O. Abikoye. She worked on pattern recognition. For details see [87] and reference therein.

One of my research areas in Computer Science is on refinement of Iris Localization Algorithm. Biometrics recognition is a common and reliable way to authenticate the identity of a living person based on physiological or behavioral characteristics. Iris recognition is one of the newer biometric recognition technologies used for personal identification. It is reliable and widely used. In general a typical iris recognition system consists of three basic module which include image acquisition, Iris Localization and pre-processing, Iris texture extraction & signature encoding and lastly Iris signature matching for recognition or verification.

One of the most important steps in iris recognition systems is iris localization, which is related to the detection of the exact location and contour of the iris in an image. (i.e it defines the inner and outer boundaries of iris region). Obviously, the performance of the identification system is closely related to the precision of the iris localization step. In this study, an efficient algorithm for iris localization is proposed. The algorithm proposed can accurately define both the inner and outer boundaries of the iris irrespective of the geometry it may be (circle or eclipse) by capturing the parameters that represents the geometry.

In the last decade computer based systems have become very essential part of our daily life. We store private information that must be secure in sense of personal or corporate cases. Since there are billions of interconnected computers world-wide, security becomes a critical problem that must be addressed by continuous new reliable and robust identification, verification or cryptographic techniques involving user – based characteristics.

Reliable automatic recognition of individuals has long been an important goal, and it has taken on new importance in recent years. Using traditional password (PIN) Personal Identification Number) or user-id systems are not secure enough to provide full access control to a system. In order to improve the security of such systems biometric information could be incorporated into the system. The use of biometric signatures, instead of tokens such as identification cards or computer passwords, continues to gain increasing attention as a means of identification and verification of individuals for controlling access to secured areas, materials, or systems. A wide variety of biometrics has been considered over the years in support of these challenges. Today, biometric recognition is a common and reliable way to authenticate the identity of a living person based on physiological or behavioral characteristics.

It has been established that physiological characteristic, such as fingerprint, iris pattern, facial feature, hand geometry, DNA etc are relatively stable physical characteristics. This kind of measurement is basically unchanging and unalterable without significant duress. A behavioral characteristic is more of a reflection of an individual's psychological makeup as signature, speech pattern, or how one types at a keyboard. The degree of intra-personal variation in a physical characteristic is smaller than a behavioral characteristic. For examples, a signature is influenced by both controllable actions and less psychological factors, and speech pattern is influenced by current emotional state, whereas fingerprint template is independent. Nevertheless all physiology-based biometrics don't offer satisfactory recognition rates (False Acceptance and/or False Reject Rates, respectively referenced as FAR and FRR).

The automated personal identity authentication systems based on iris recognition are reputed to be the most reliable among all biometric methods. The use of iris recognition for identification and verification is crucial because it is considered as a reliable solution in establishing a person's identity. Iris recognition is forecast to play a role in a wide range of other applications in which a person's identity must be established or confirmed. These include electronic commerce, information security, entitlements authorization, building entry, automobile ignition, digital forensic and police applications, network access and computer applications, or any other transaction in which personal identification currently relies just on special possessions or secret (keys, cards, documents, passwords, PINs).

The probability of finding two people with identical iris pattern is almost zero. That's why iris recognition technology is becoming an important biometric solution for people identification in access control as networked access to computer application. Compared with other biometric signatures mentioned above, the iris is generally considered more stable and reliable for identification.

One of the most important steps in iris recognition systems is iris localization, which is related to the detection of the exact location and contour of the iris in an image. (i.e it defines the inner and outer boundaries of iris region) Obviously, the performance of the identification system is closely related to the precision of the iris localization step. Some previous iris segmentation approaches assume that both inner and outer boundaries of iris as a circle, while some assume that the boundaries can be modeled as both eclipse and circle respectively.

To improve the quality of segmentation /localization (i.e inner and outer boundaries of iris region), the researchers propose a new method based on development of an algorithm that uses the concept of Cartesian co-ordinate system and the use of the method for the computation of radii which are subsequently used to outline the inner and outer boundaries of the iris thus localizing it. The parameters captured to represent the ellipse are represented as objects. The new algorithm is simple, stable, robust, compute fast and flexible.

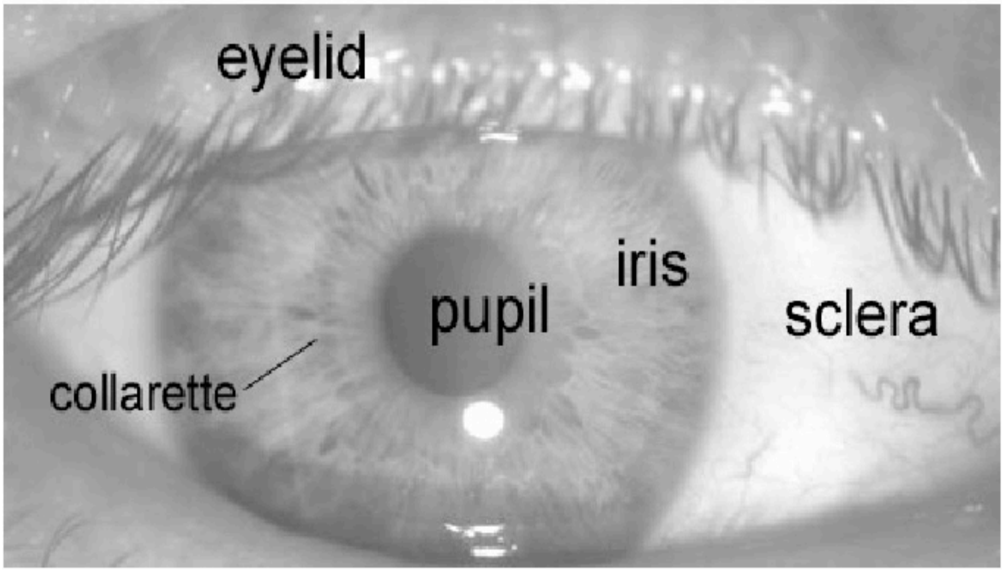


Figure 1: Anatomy of a frontal image of the eye [87].

**The Human Iris**

The iris is a muscle within the eyes that regulates the size of the pupil, controlling the amount of light that enters the eye. It is the colored portion of the eye with coloring based on the amount of melatonin pigment within the muscle. Melatonin pigment is a hormone produced by the pineal gland that affects the physiological changes related to time/lighting cycles. The iris is also an annular area between the pupil and the white sclera in the eye; it has many interlacing features such as stripes, freckles, coronas, radial furrow, crypts, zigzag collarette, rings etc collectively referred to as texture of the iris (see Figure 1). This texture is well known to provide a signature that is unique to each subject. In fact, the operating probability of false identification by the Daugman algorithm [5] can be of the order of 1 in $10^{10}$.

The Iris lies between the cornea and the lens of the human eyes. The iris image is shown in figure 1. The iris is perforated close to its center by a circular aperture. This is known as Pupil. The pupil size can vary from 10% to 80% of the iris diameter. The iris diameter is 12mm.

The Human iris consisted of three boundaries. The boundaries are shown in figure 2. The first boundary is the inner boundary which lies between the pupil and the iris. The pupil also has a low grey level and looks dark in the eyes image. The second boundary is an outer boundary. The outer boundary is between the iris and the sclera and the last boundary is the collarette boundary.

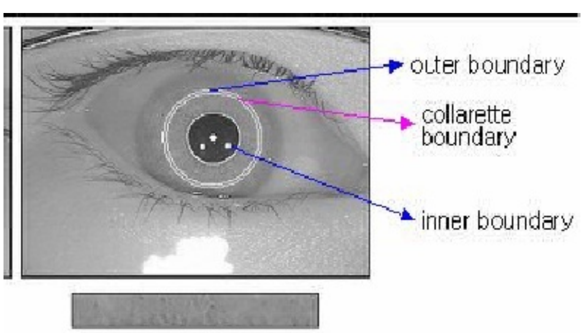


Figure 2: Image of the boundaries and the localization iris area [87].

**PREVIOUS WORK ON IRIS LOCALIZATION**

Daugman's algorithm

Daugman introduced a circular edge detection operator for iris localization, as follows:

$$\max_{(r,x_0 y_0)} \left| G_\sigma(r) * \frac{\partial}{\partial_r} \oint_{r,x_0 y_0}^{s} \frac{I(x,y)}{2\pi r} ds \right| \qquad (1)$$

operators that search over the image domain (*x*,*y*) for the maximum in the blurred partial derivative, with respect to increasing radius *r*, of the normalized contour integral of *I*(*x*,*y*) along a circular arc *ds* of radius *r* and center coordinates ($x_o$,$y_o$). The symbol * donates convolution and G(r) is a Gaussian filter used as a smoothing function. It is obvious that the results are inner and outer boundaries of iris. First, the inner boundary is localized, due to the significant contrast between iris and pupil regions. Then, outer boundary is detected, using the same operator with different radius and parameters.

Hough Transform

Wildes [87], Kong and Zhang [87] and Ma et al. [87] use Hough transform to localize irises. It uses the gradient-based Hough transform to decide the two circular boundaries of an iris. It includes two steps. First a binary edge map is generated by using a Gaussian filter. Then, votes in a circular Hough space are analyzed to estimate the three parameters of one circle $(x_0, y_0, r)$. A Hough space is defined as:

$$H(x_0, y_0, r) = \sum_i H(x_i, y_i x_0, y_0, r) \qquad (2)$$

Where:

$(x_i, y_i) \qquad = An\ edge\ pixel$

$$H(x_i, y_i, x_0, y_0, r) = \begin{cases} 1\ if\ (x_i, y_i) is\ \ on\ the\ circle\ (x_0, y_0, r) \\ 0\ otherwise \end{cases}$$

The location$(x_0, y_0, r)$ with the maximum value of $H(x_o, y_o, r)$is chosen as the parameter vector for the strongest circular boundary.

Other Localization Methods

Other researchers use methods similar to the described segmentation methods. Tisse et al. proposed a segmentation method based on Integro-differential and the Hough transform. Huang et al. proceeded to iris segmentation by simple filtering, edge detection and Hough transform. Boles and Boashah and Lim et al. mainly focused on the iris image representation and feature matching without introducing a new method for segmentation. Noh et al also did not propose a new method for segmentation, Daugman Algorithm was utilized. Tan et al [87] proposed a segmentation method based on canny operator and Hough transform. Tang et al also proposed a new segmentation method based on Support Vector Machines (SVM) classifier to locate the iris outer boundary and Dilation operator of morphological to locate the iris inner boundary. Gupta et al [87] used Hough transform and circular summation of intensity approach. Yi et al [87] used canny edge detection for its iris segmentation. Also Du et al [87] proposed the iris detection method based on the prior pupil segmentation. Yahaya et al [87] proposed a segmentation method based on direct least square fitting of ellipse,Integro-differential algorithm and also Hough Transform.

Considering the above mentioned methods, we can state the following remarks:

1. In Daugman algorithm, Hough transform, usually inner boundaries are detected by circle fitting techniques. This is a source of error, since the inner boundaries are not exactly circles [87].
2. Computationally, they are tedious. Especially, Wildes method is very computationally demanding because it introduces lots of edge points of other objects, such as eyelashes and eyelids, in Hough transform [87]
3. They are lacking in robustness.

**ALGORITHM FOR IRIS LOCALIZATION**

The methodology used is the development of an algorithm that captures the Cartesian coordinates of the center of the eye image and two other points for each of the boundaries(inner and outer). These points are used to compute the radii of the boundaries and then used to localize the iris by drawing a perfect geometry that fits the boundaries. The center points and the computed radii serve as the parameters which are stored as objects- one for the inner boundary and another for the outer boundary. The steps involved in the iris localization process are summarized as follows:

Step 1: Create Object1, Object2 (Objects are composite data types that stores the parameters representing the geometry for the boundaries of the iris, the parameters include x,y, r1,r2 where x, y are co-ordinates of the center of the eye image and radius1, radius2 respectively)

Step 2: Capture the co-ordinate of the centre of the image of the eye.

Step 3: Capture the co-ordinates of 2 points on the inner boundaries of the iris ( one of the 2 points is vertically above the centre of the eye and the other is horizontally eastward of the centre of the eye i.epX, pY respectively)

Step 4: Compute:
radius1 (r1) as distance between the centre of the eye image and the vertical points co-ordinates.

$$r1 = \sqrt{x_1^2 + y_1^2} \qquad (3)$$

Where $x_1 = pX.x_1 - pcentre.x_1$
$y_1 = pX.y_1 - pcentre.y_1$

radius2 (r2) as distance between the center of the eye image and the horizontal eastward co-ordinates.

$$r2 = \sqrt{x_2^2 + y_2^2} \qquad (4)$$

Where $x_2 = pY.x_2 - pcentre.x_2$
$y_2 = pY.y_2 - pcentre.y_2$

Step 5: Draw an Eclipse using the co-ordinates of the centre of the eye, radius1 and radius2 (r1 and r2).

Step6: Repeat step 3 to 5 for the outer boundary.

Step 7: Store the co-ordinates of the centre of the eye image and the 2 computed radius for the inner boundary into Object1.

Store the co-ordinates of the centre of the eye image and the 2 computed radius for the outer boundary into Object2.

Step 8: Stop.

**Our result**

One of the most important steps in iris recognition systems is iris localization, which is related to the detection of the exact location and contour of the iris in an image. (i.e it defines the inner and outer boundaries of iris region) Obviously, the performance of the identification system is closely related to the precision of the iris localization step. Some previous iris segmentation approaches assume that both inner and outer boundaries of iris as a circle, while some assume that the boundaries can be modeled as both eclipse and circle respectively. Usually, the inner boundaries are detected by circle fitting techniques. This is a source of error, since the inner boundaries are not exactly circles. In noisy situations, the outer boundary of iris does not have sharp edges.

In this research work the algorithm developed can accurately define both the inner and outer boundaries of the iris irrespective of the geometry it may be (circle or eclipse) by capturing the parameters that represents the geometry. The new algorithm is simple, robust, computes accurately and flexible.

### d. FUZZY LOGIC

In real life situation, weare used to the classical two valued logic of either yes or no, truth or false, on or off, good or bad etc. We tend to neglect the middle values. In classical set theory it is a question of an element been a member of a set or not (complement). Fuzzy logic is an area of research, which provides a solution to the problems of vagueness which departs from the **all** or **nothing** logic. It logically redefines yes or no ideas in proper form. Fuzzy sets were proposed to deal with vagueness related to the way people sense things ( e.g Tall versus short, big versus small). A set is defined by its elements and the membership of each element in the set.

#### FUZZY SET OPERATION

Union

The membership function of the Union of two fuzzy sets A and B with membership functions $\mu_A$ and $\mu_B$respectively is defined as the maximum of the two individual membership functions. This is called maximum criterion.

$\mu_{AUB}$= max($\mu_A$, $\mu_B$).

The Union operation in fuzzy set theory is the equivalent of the OR operation in Boolean algebra.

**INTERSECTION**

The membership function of the intersection of two fuzzy sets A and B with membership functions $\mu_A$ and $\mu_B$ respectively is defined as the minimum of the two individual membership functions. This is called minimum criterion.

$\mu_{AnB}$= min($\mu_A$, $\mu_B$)

The Intersection operation in Fuzzy set theory is the equivalent of the AND operation in Boolean algebra.

**COMPLEMENT**

The membership function of the complement of fuzzy set A with membership function is defined as the negation of the specified membership function. This is called the negation criterion. $\mu_{\bar{A}}$= 1-$\mu_A$

The complement operation in Fuzzy set theory is the equivalent of the NOT operation in Boolean algebra.

The following rules, which are common in Classical set theory, also apply to Fuzzy set theory: De Morgan's law, Associativity, Commutativity and Distributiivity

Systems undergo three transformation to become system input viz:

**Fuzzification, Rulebase and Defuzzification process**

(i) Fuzzification: This is a process that uses predefined membership functions that maps each system input into one or more degree of membership(s).

(ii) Rulebase: Rule (Predefined) is evaluated by combining degrees of membership to form output strengths.

(iii) Defuzzification: This is a process that computes system outputs based on strengths and membership functions.

**MODEL DEVELOPMENT**

In general, the membership function of fuzzy subset $A_{\mu A}$in the universal set X is defined as $\mu_A$[0,1], where [0,1] is the interval of real numbers with 0 and 1 being the minimum and maximum membership values respectively.

The two most popular Defuzzification methods are the Mean-of-Maximum (MOM) and the Centre of Area (COA) methods. For MOM, the crisp output Δq is the mean value of all points whose membership values are maximum. In the case of discrete universal set W, MOM is defined by Zimmermann 1987:

$$X=\sum_{i=1}^{n}\frac{\omega_i}{n}$$

Where $\{\omega_i:\mu_c(\omega_j), \omega_l, \omega_j \in W, \omega_i \neq \omega_j\}$ and n is the number of such support values. As for COA, the crisp output Δq is the centre of gravity of distribution of membership functions $\mu_c$. In the case of discrete universal set W, COA is defined as Zimmermann 1987b:

$$X=\frac{\sum_{i=1}^{n}(\mu c(w_i)x(w_i))}{\sum_{i=1}^{n}\mu c((w_i))}$$

Where n is the number of elements of the fuzzy set C and $\omega \in W$.

In this model, the COA method is used for Defuzzification because the MOM method has the worst performance with respect to both accuracy and convergence speed. Therefore, the COA method yields better results than MOM.

Fuzzy logic is a superset of convectional (Boolean) logic that has extended to handle the concept of partial truth, that is, true values between completely true and completely false [Zadeh (1965)] uses the interval value between 0 (false) and 1 (Truth) to describe any human reasoning.

The logic requires knowledge in order to reason and this knowledge which is provided by an expert who knows the process is stored in the Fuzzy System [Zimmerrnann (1987)].

It is used in artificial intelligence that deals with the reasoning. These are used in applications where process data cannot be represented in binary form.

It is being applied in knowledge based automatic decision making [Zimmerrnann (1991)], forecast evaluation [Lee (2005)] and processes control engineering [Sugemo (1985)]. A concept of a fuzzy integral was defined by [Sugemo (1985)].

Ralescu and Adams (1980) define the fuzzy integral of a positive, measurable function, with respect to a fuzzy measure and show that the monotone convergence theorem and Fatou's lemma coincides with fuzzy integral. In particular, they established the convergence theorem is in a way stronger than the lebesque-dominated convergence theorem. Our main goal in this paper is, therefore, to establish some basic theorem on integral of fuzzy sets using the fuzzy measures which are applicable in various areas of life [Rudin, (1974)] Professionals in the field of Education have always considered factors responsible for students' performance in examination-good or bad- but have not been able to draw a clear conclusion on how to maintain a desirable performance or forestall an undesirable one. An important part of this work is how to improve on the quality of future performance of students based on their previous performance using fuzzy model approach.

Fuzzy logic was introduced by Zadeh (1965) and used by Sugeno (1985) to control dynamic system. Since then, this has been used successfully in so many areas, especially in the area of control and decision making. See, Zimmerrnann (1987), Zimmerrnann*etal*(1987), Peng*et al*(2002) and Omolehin*et al* (in press)

**Deterministic Model**

This approach is widely used by Educationists to measure students' performance in an examination. In this method, the overall performance of students is evaluated by summing the averages of the individual performance in a set.

**Fuzzy Model**

The fuzzy model suggested by Sugeno (1985) represents a mathematical tool, which is used to build a fuzzy model of a system. This consists of a set of implication rules, which are used to express control statements. An implication rule contains fuzzy variables with unimodal membership functions. Such membership functions are linguistic variables. For further information see Oyelade (2004). I have worked and published many papers in Fuzzy logic.

## 8. SOME AREAS OF APPLICATION OF MY RESEARCH

### a. Presidential Election of 1979 ( $12\frac{2}{3}$ issue)

An irreparable injury was done to mathematics in our dear country, Nigeria during the presidential election that took place in 1979. The problem then was to determine what constitutes two third of 19 states. The two leading candidates then were Alhaji Shehu Shagari and Late Chief Obafemi Awolowo. Alhaji Shagari was declared the winner after winning 12 states and $\frac{1}{4}$of the $\frac{2}{3}$ of the total votes cast in Kano State (the 13$^{th}$ state). The Chairman of the then Federal Electoral Commission (FEDECO), Late Justice Ovie Wisky based his declaration of Shagari the winner on Late Chief Akinjide's interpretation of $\frac{2}{3}$ of 19 states.

The Presidential Tribunal's main task was in the interpretation of the provision of Section 34A (1)(c)(ii) of the Electoral Decree 1977 No. 73 which provides – "34A (i) A candidate for an election to the office of President shall be deemed to have been duly elected to such office where – .. (c) there being more than two candidates – (ii) he has not less than one-quarter of the votes cast at the election in each of at least two-thirds of all the States in the Federation."

The constitution is very clear. There is no ambiguity in its intention. The word 'at least' there is very significant in mathematical interpretation of that section. That is if we cannot get the exact value we accept the next one greater than it in the domain of the definition. This is an integer programming problem.

In optimization, problems are grouped into classes with appropriate algorithm designed for each class. This particular problem falls under integer programming class in optimization. State is an entity and we cannot have a fractional state. Since the domain of the problem is 19 states, we have two choices as follows:

i. Fortran Programming (integer variable ) $NS = \frac{1}{3} x\ 19$ Answer=12 states
ii. Infimum and Supremum principle (Least upper integer) Answer=13 states

Because of the word 'at least $\frac{2}{3}$ ', clearly 12 states is not in the visible region. The visible region is 13 to 19 and hence, the visible solutions are 13, 14, 15, 16, 17, 18 and 19 states. Therefore the candidate ought to have scored $\frac{1}{4}$ in at least 13 states to avoid a run-off. **Certainly not in 12 states**. It was an irreparable damage done to mathematics and it should be avoided in future.

**b. Fuzzy Approach to Corruption**

Corruption is a crime against humanity and it has eaten deep in to the fabric of Nigeria society. All Nigerians must join hands together to fight the menace. A revolutionary step proposed here is to make a law on **fuzzy scale** and the center of gravity of the decision table be made death penalty. It will curb corrupt practices. Our money that have been stolen could have been used to develop our youths instead of castigating them of been illiterates. Actually it is the political class that is very lazy, not the youths. They cannot fix our economy because they are not creative. All they can do is to embezzle our money. They lack initiatives. I hereby propose that the legislative business in Nigeria be done on part-time bases with only one arm legislature. The current dispensation is parasitic on our National treasury.

**Fuzzy Algorithm for Corruption**

A model of an algorithm to fix corrupt practices in Nigeria is here proposed in this section. We, in Mathematical Sciences believe that anything can be improved upon. Therefore, the model will be a modified version of **PMB** method, which has many challenges. The new method will be called **Nigeria Corruption Fixer Method (NCFM)** algorithms.

Steps involved in **NCFM** algorithm are as follows:

**Description:** It is going to be a mapping or functions defined on a fuzzy scale in the interval [0, 1). The domain of the mapping is the level of corruption. This level of corruption is fuzzified and it will map the appropriate punishment in co-domain. It will also have death penalty in its center of gravity. The proposed algorithm will be robust and it will reduce short comings of the known methods to the barest minimum.

I hereby appeal to the Federal Government to make grant available for my research group **(OmoConsultSoft)** to produce a trail version of the proposed **of NCFM.**

**c. Remover of a Senator as contained in Nigeria constitution**

Section 69 of the 1999 Constitution provides 10 clear steps on how a serving senator can be recalled from the Senate by his constituency.

The steps are:

i. More than half of the registered voters in the Senator's senatorial district write, sign and send a petition to the Chairman of the Independent National Electoral Commission, INEC alleging their loss of confidence in the senatorial

ii. The petition must be signed, and arranged according to polling units, wards, Local Government Areas, and constituency.

iii. INEC notifies the Senator sought to be recalled, stating that it has received a petition for his or her recall, if the petition is valid

iv. INEC issues a public notice or announcement stating the date, time and location of the verification of signatures to the petition

v. INEC verifies the signatures to the petition at the designation. The signatories must be individuals who appear on the voters' register.

vi. INEC conducts a referendum if more than one half (50% + 1) of the signatories are verified

vii. INEC writes to the petitioners stating that the minimum requirements for a referendum were not met, if the number verified is less than one half of the registered voters in that constituency. The petition will therefore be dismissed.

viii. INEC conducts a referendum within 90 days of receipt of the petition if the minimum requirements for a referendum are met. The referendum will be a simple yes or no vote on whether the Senator should be recalled, and will be decided by simple majority of the votes of the persons registered to vote in that Senator's constituency.

ix. If majority of the voters in the constituency vote 'yes' the Chairman of the INEC will send a Certificate of Recall to the Senate President to effect the recall.

x. The Senate President will show affected senator the way out of the Senate.

Item number (vi) should be corrected to read "INEC conducts a referendum if more than fifty per cent plus one (50% + 1) of the signatories are verified". This is to avoid ambiguity and misinterpretation in case the number is odd.

## 9. Conclusion

Madam Vice Chancellor, Ma, I have just briefly explained my areas of research and I have revealed some applications of my research findings. It is clear that Mathematics can solve complicated problems hence; Mathematics must be given adequate attention it deserved. In most advanced countries Mathematicians form the nucleus of their employment in to their security outfits because of their reasoning capabilities. Nigeria should borrow a leave from those countries by employing Mathematicians to form a larger percentage of its security agencies.

In future, Mathematicians must be consulted before taking decisions on major constitutional issues so that damages will not be done to Mathematics and polity as it was done during the saga of $12\frac{2}{3}$. in 1979.

I hereby appeal to relevant authority to implement all recommendations contained in this lecture. In so doing it will be the beginning of Mathematical revolution in this country.

## 10. Recommendations

The following steps are recommended to the stake holders in Mathematics for immediate implementation:

- Local seasoned Mathematicians should be encourage to write Mathematics text books with local contents as much as possible;
- Authors of mathematics text book should make their books self-explanatory like most Indian Authors;
- Enlightenment campaign should be vigorously mounted on the aspect of importance and applications of mathematics;
- Government and cooperate body should always make yearly budgetary for mathematical sciences and education;
- People with Mathematics with education should be employed to teach Mathematics at all levels. The practice of making people with other qualifications (i.e. physics, chemistry, biology, economics, e.t.c.) to teach Mathematics should be abolished;
- Teachers of Mathematics at tertiary institutions must be sponsored for PGDE;
- Outlet for information dissemination on new discoveries in Mathematics should be established;
- Provision of research grant must be made and easily accessible;
- Automatic scholarship must be given to anybody offering Mathematics at postgraduate level in the University;
- Internet facilities and other electronic print media for Mathematics should be made available at cheaper, friendly and avoidable at rural an urban centers;
- Modern text books and instructional materials should be made available. Any imported materials on Mathematics should be import duty free;
- Mathematics should be taught everyday in Primary and Secondary levels;

- Motivational boosting mechanism must be permanently institutionalized and
- National Mathematical Centre must be well funded to be in the fore front of the implementation of these recommendations.

Any nation that does not pay adequate attention to its mathematical program will be destined to be a consumer nation. Its technological advancement will be a mirage. We are in the era of technological competitive word.

Madam Vice Chancellor Ma, if all these recommendations are implemented **MATHEMATICAL FIXATION ALGORITHM FOR CORRUPTION** will a formidable tool as a problem solver.

## 11. Acknowledgement

I bestow all Honour, glory and Adoration to God Almighty the greatest who made it possible for me to be alive to give this inaugural lecture. I bless God for making me to reach the apex of the profession that He has given me which I love so much. I became a Professor of Mathematics by the divine power of God Almighty. It is neither by my power nor by my wisdom.

I thank my parent; my late father, His Royal Highness Obajana Omolehin, my late mother Olori Adehoro Omolehin for all they did for me to have education despite the fact that they were not opportune to have formal education.

My gratitude goes to my brothers and sisters who have helped me in one way or the other. In particular I thank my late brothers Mallam Kolawole Bello (a.k.a. Shugaba) and Nurse. R. O. Omolehin who were responsible for my primary education

I acknowledge and appreciate my uncles namely Mallam Yusuf Bakare and Alhaji (Chief) D. O. Bakare for the roles they played in my life after my primary education. God Almighty will reward you.

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